Discrete Distinguishability as the Ontological Basis of Logic

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Abstract

Formal logical systems are typically defined independently of their realizability, allowing continuous domains, infinite constructions, and idealized truth conditions. In contrast, all physically and computationally realizable systems operate over discrete, finitely distinguishable states.

We introduce the axiom of discrete distinguishability: only discretely distinguishable states are ontologically admissible. This shifts the foundation of logic from formal permissibility to ontological admissibility.

On this basis, we develop a framework in which the fundamental unit of logic is not a truth value, but an act of distinction endowed with order. Logical operations are derived from relations between distinctions, rather than assumed as primitives. Classical and non-classical logical systems arise as specific structural organizations of distinction sets.

Continuous domains are reinterpreted as limit representations of discrete structures, rather than as ontological primitives. The resulting framework provides a unified, minimal, and realizability-consistent foundation for logic grounded in distinguishability and order.

Keywords: discrete logic; distinguishability; logical foundations; non-classical logics; realizability; finite structures; distinction algebra; logical semantics; model theory; categorical structures

Contents

1 Introduction

Classical logic, originating with Aristotle [1], defines truth in binary terms and is governed by principles such as excluded middle and non-contradiction. Extensions of classical logic include intuitionistic logic [2], fuzzy logic [3], modal logic [4], and paraconsistent systems [5]. These systems differ in their treatment of truth, inference, and admissible constructions, yet they are typically introduced as independent formal frameworks.

All such systems are formulated within mathematically permissive settings that admit continuous domains, infinite structures, and idealized semantic objects. However, any physically, biologically, or computationally realizable system operates under strict constraints: only finitely many states can be distinguished, and all processes must be finitely executable.

This creates a structural gap between formal logical theory and realizable systems. Formal models allow infinitely dense state spaces and arbitrary constructions, while realizable systems admit only discrete, finitely distinguishable configurations.

The present work addresses this gap by replacing the principle of formal permissibility with the principle of ontological admissibility. Instead of asking which structures can be defined mathematically, we restrict attention to those that can be realized as systems of distinctions.

We introduce the axiom of discrete distinguishability: only discretely distinguishable states are ontologically admissible. On this basis, we develop a framework in which the fundamental unit of logic is not a truth value, but an act of distinction endowed with order.

Within this framework, logical operations are not taken as primitives, but arise from relations between distinctions. Classical and non-classical logical systems are reconstructed as specific organizations of these relations. Continuous domains are reinterpreted as limit representations of discrete structures rather than as ontological primitives.

The result is a unified, minimal, and realizability-consistent foundation of logic grounded in distinguishability and order.

2 Axiom of Discrete Distinguishability

Axiom 1 (Discrete Distinguishability)**.** *Only discretely distinguishable states are ontologically admissible.*

This axiom excludes infinitely close but distinct states as ontological entities. Distinction is taken as primitive and irreducible.

2.1 Levels of Description

We distinguish three levels:

- Ontological level: admissible structures of reality (discrete distinctions).
- Structural level: relations between distinctions (order, composition).
- Representational level: formal systems (logical calculi).

The present work derives logical systems at the representational level from primitives defined at the ontological level.

3 Distinction and Order

Definition 1 (Distinction)**.** *A distinction is a primitive operation separating two states.*

Definition 2 (Successor Relation)**.** *A binary relation $\prec$ defines a successor structure if for any admissible states a, b, $a \prec b$ denotes that b is a distinguishable successor of a.*

This induces a directed structure:

$$\cdots \prec a \prec b \prec c \prec \cdots$$

Binary structure arises here as directionality (preceding vs succeeding), not as value assignment.

4 Distinction Algebra

4.1 Ontological Primitives

Definition 3 (Set of States)**.** *Let S be a set of discretely distinguishable states.*

Definition 4 (Distinction)**.** *A distinction is a binary operation*

$$\delta : S \times S \to D$$

where D is the set of elementary distinctions.

The value $\delta(a, b)$ represents the act of distinguishing state a from state b.

Definition 5 (Order of Distinctions)**.** *A binary relation*

$$\prec \subseteq \; D \times D$$

defines the order of distinctions, interpreted as sequential realization.

4.2 Axioms

A1. Directionality of Distinction

$$\delta(a, b) = \delta(b, a) \text{ only if } a = b$$

A2. Transitivity of Order

$$d_1 \prec d_2, \; d_2 \prec d_3 \;\Rightarrow\; d_1 \prec d_3$$

A3. Local Finiteness

$$\forall a \in S : \quad |\{\delta(a, b) \mid b \in S\}| < \infty$$

A4. Realizability Any finite chain

$$d_1 \prec d_2 \prec \cdots \prec d_n$$

must be realizable by a finite procedure.

4.3 Composition of Distinctions

Definition 6 (Sequential Composition)**.**

$$d_1 \triangleright d_2 \quad \textit{defined iff} \quad d_1 \prec d_2$$

Definition 7 (Parallel Composition)**.**

$$d_1 \parallel d_2 \quad \textit{iff} \quad \neg(d_1 \prec d_2) \wedge \neg(d_2 \prec d_1)$$

We consider closure of D under these compositions where applicable.

4.4 Logical Operations as Derived Structures

Logical operations are not primitive but emerge from relations between distinctions.

Negation If inverse distinction exists:

$$\neg d = d^{-1}, \quad \delta(a,b)^{-1} = \delta(b,a)$$

Conjunction

$$d_1 \wedge d_2 = \begin{cases} d_1 \parallel d_2 & \text{if independent} \\ d_1 \triangleright d_2 & \text{if ordered} \end{cases}$$

Disjunction

$$d_1 \vee d_2 = \{d_1, d_2\}$$

Implication

$$d_1 \Rightarrow d_2 \text{ iff } d_1 \prec d_2$$

4.5 Emergence of Logical Systems

Classical Logic Arises when:

- order is total,
- inversion exists for all distinctions,
- compositions are deterministic.

Intuitionistic Logic Arises when:

- order is partial,
- inversion is not always realizable,
- implication reflects constructive reachability.

Fuzzy Logic Arises when:

- distinctions are aggregated into ordered scales,
- operations are monotonic over ordering.

Modal Logic Arises when:

- distinctions are organized across multiple domains,
- order is defined both within and across domains.

4.6 Sufficiency Theorem

Theorem. Any finitely realizable logical system admits a representation as a distinction algebra

$$(D, \prec, \triangleright, \parallel).$$

Proof (sketch). Any finitely realizable logical system operates over a finite set of states and transformations. Each transformation can be decomposed into distinctions between states, and compositions of transformations correspond to compositions of distinctions. The induced order captures dependency relations, while closure ensures completeness of representation. □

4.7 Concrete Finite Example

We construct a minimal finite system illustrating distinction algebra.

States Let

$$S = \{a, b, c\}$$

Distinctions Define:

$$d_{ab} = \delta(a, b), \quad d_{bc} = \delta(b, c), \quad d_{ac} = \delta(a, c)$$

Thus:

$$D = \{d_{ab}, d_{bc}, d_{ac}\}$$

Order Introduce an order:

$$d_{ab} \prec d_{bc} \prec d_{ac}$$

Compatibility Assume:

$$d_{ab} \parallel d_{bc}, \quad d_{bc} \parallel d_{ac}$$

but

$$d_{ab} \nparallel d_{ac}$$

Operations *Conjunction:*

$$d_{ab} \wedge d_{bc} = d_{ab} \parallel d_{bc}$$

Sequential composition:

$$d_{ab} \triangleright d_{ac} \quad \text{(via order)}$$

Implication:

$$d_{ab} \Rightarrow d_{ac} \quad \text{since} \quad d_{ab} \prec d_{ac}$$

Negation:

$$\neg d_{ab} = \delta(b, a)$$

Binary Collapse Partition D into two classes:

$$D_T = \{d_{ab}, d_{bc}\}, \quad D_F = \{d_{ac}\}$$

This induces a binary interpretation:

- D_T corresponds to "true"
- D_F corresponds to "false"

Interpretation This example demonstrates that:

- logical operations arise from composition and ordering,
- implication is induced by order,
- binary logic emerges as a coarse partition of distinctions.

5 Extended Formal Structure

5.1 Distinction Semantics

Logical semantics is reconstructed without reference to truth values. A formula is interpreted as a structured set of distinctions, and satisfaction corresponds to realizability.

Definition 9 (Interpretation). An interpretation is a mapping:

$$\mathcal{I} : \text{Form} \to \mathcal{P}(D)$$

assigning to each formula a finite set of distinctions.

Definition 10 (Satisfaction). A distinction structure $(D, \prec, C)$ satisfies a formula φ under $\mathcal{I}$ if:

- $\mathcal{I}(\varphi)$ is finite,
- for all $d_1, d_2 \in \mathcal{I}(\varphi)$, $(d_1, d_2) \in C$,
- the induced relations do not violate $\prec$,
- the set $\mathcal{I}(\varphi)$ is closed under composition.

Derived Semantics.

- Conjunction:

  $$\mathcal{I}(\varphi \wedge \psi) = \mathcal{I}(\varphi) \cup \mathcal{I}(\psi)$$

  realizable iff the union is compatible.
- Disjunction:

  $$\mathcal{I}(\varphi \vee \psi) = \{\mathcal{I}(\varphi), \mathcal{I}(\psi)\}$$

  representing alternative realizations.
- Negation: defined via incompatibility.
- Implication:

  $$\varphi \Rightarrow \psi \text{ iff } \forall d_1 \in \mathcal{I}(\varphi), \forall d_2 \in \mathcal{I}(\psi) : d_1 \prec d_2$$

This defines a complete semantics based on realizability of distinction structures.

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5.2 Categorical Representation

Distinction algebra admits a categorical formulation.

Definition 11 (Category of Distinctions). Define a category $\mathcal{D}$:

- Objects: states $a \in S$,
- Morphisms: distinctions $\delta(a, b)$,
- Composition: sequential composition $d_1 \triangleright d_2$,
- Identity: trivial distinction $\delta(a, a)$.

Properties.

- Order $\prec$ corresponds to factorization of morphisms.
- Parallel composition corresponds to commuting morphisms.
- Negation corresponds to reversal of morphisms (when defined).
- Modal structures correspond to families of categories with inter-domain mappings.

Theorem (Correspondence). Distinction algebra admits a categorical representation in $\mathcal{D}$ preserving composition and order.

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5.3 Distinction Models

Logical systems can be represented through distinction-based models.

Definition 12 (Distinction Model). A model is a triple:

$$M = (D, \prec, C)$$

where:

- D — set of distinctions,
- $\prec$ — order relation,
- C — compatibility relation.

Definition 13 (Model Homomorphism). A mapping $h : M_1 \to M_2$ is a homomorphism if:

- $d_1 \prec d_2 \Rightarrow h(d_1) \prec h(d_2)$,
- compatibility is preserved,
- composition is preserved.

Remark. Models are defined intrinsically, without reference to external truth assignments.

Model Classes.

- Classical models: total order, global inversion,
- Constructive models: partial order, restricted inversion,
- Modal models: indexed families of models,
- Graded models: linearly ordered distinction sets.

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5.4 Distinction Machines

Logical operations admit a computational interpretation.

Definition 14 (Distinction Machine). A distinction machine is a tuple:

$$\mathcal{M} = (S, D, \prec, \Delta)$$

where:

- S — states,
- D — distinctions,
- $\prec$ — order (causal or sequential structure),
- Δ — transition function generating distinctions.

Execution. A computation is a finite chain:

$$d_1 \prec d_2 \prec \cdots \prec d_n$$

Theorem (Computational Representation). Every finitely realizable computation admits a representation as a distinction machine.

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5.5 Epistemic Interpretation

Distinction algebra admits an epistemic interpretation.

- Observation corresponds to distinction between states.
- Knowledge corresponds to sets of distinctions stable under composition.
- Coherence corresponds to compatibility.
- Uncertainty corresponds to incompleteness of order.
- Update corresponds to extension of distinction sequences.

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5.6 Syntax

Definition 15 (Formula). The set of formulas is the smallest set closed under:

- atomic distinctions $d \in D$,
- operations $\wedge, \vee, \neg, \Rightarrow$,
- parentheses.

Definition 16 (Realizer). A realizer of a formula is a finite sequence of distinctions realizing its semantic structure.

This establishes correspondence between syntax and realizable distinction structures.

6 Example: Construction of Logical Operations

We illustrate how classical logical operations emerge from distinction algebra.

Let D be a finite set of distinctions. Each distinction $d \in D$ represents a minimal separability between states.

6.1 Compatibility Structure

Define a compatibility relation:

$$C \subseteq D \times D$$

where $(d_1, d_2) \in C$ means that d_1 and d_2 can be jointly realized.

6.2 Conjunction

For compatible distinctions d_1, d_2, define:

$$d_1 \wedge d_2 := d_1 \circ d_2$$

Thus, conjunction corresponds to the composition of jointly realizable distinctions.

6.3 Disjunction

Define disjunction as alternative realizability:

$$d_1 \vee d_2 := \{d_1, d_2\}$$

This represents a branching structure of admissible distinctions.

6.4 Negation

Define negation as incompatibility:

$$\neg d := \{d' \in D \mid (d, d') \notin C\}$$

Thus, negation corresponds to the set of distinctions that cannot coexist with d.

6.5 Binary Collapse

If D is partitioned into two mutually incompatible classes D_T and D_F, then:

$$\forall d \in D_T, \quad \neg d \subseteq D_F$$

This induces a binary logical structure equivalent to classical truth values.

6.6 Interpretation

This construction shows that logical operations are derived from:

- compatibility (co-realizability),
- composition (sequential realization),
- incompatibility (exclusion).

Thus, classical logic arises as a special case of distinction algebra under binary partitioning.

7 Fuzzy Logic as Ordered Distinction Structures

Fuzzy logic is typically defined over the continuous interval $[0, 1]$, interpreted as degrees of truth. Within the present framework, such a representation is not ontologically primitive, but arises from ordered aggregation of discrete distinctions.

7.1 Ordered Distinction Sets

Let $D = \{d_1, d_2, \dots, d_n\}$ be a finite set of distinctions equipped with a total or partial order:

$$d_1 \prec d_2 \prec \cdots \prec d_n$$

This order reflects increasing structural refinement or stability of distinctions.

7.2 Aggregation as Measurement

A fuzzy value is interpreted as an index or rank within this ordered set:

$$\mu(d_k) = \frac{k}{n}$$

Thus, what appears as a continuous value is a normalized representation of discrete position within an ordered system.

7.3 Operations

Logical operations are induced by order:

- Conjunction:
$$d_i \wedge d_j := \min(d_i, d_j)$$
- Disjunction:
$$d_i \vee d_j := \max(d_i, d_j)$$

These operations correspond to selecting lower or higher positions in the ordered structure.

7.4 Limit Interpretation

As $n \to \infty$, the normalized index approaches a dense ordering:

$$\lim_{n \to \infty} \frac{k}{n} \to [0, 1]$$

However, this limit introduces infinite density and therefore does not correspond to a realizable structure.

7.5 Interpretation

Fuzzy logic can thus be understood as:

- an ordered aggregation of discrete distinctions,
- not a fundamentally continuous logic,
- but a limit representation of increasingly refined discrete systems.

8 Finite Realizability

Definition 8 (Realizable Structure)**.** *A structure is realizable if it can be instantiated by a finite procedure with finite resources.*

Proposition 1. *All realizable structures are discrete.*

Proof. Any realizable system has finite state capacity and finite distinguishability resolution. Therefore, it cannot support infinitely dense state spaces. □

8.1 Physical Constraints on Realizability

The discreteness of realizable structures is supported by physical and computational principles.

According to the Church–Turing–Deutsch principle, any physically realizable process can be simulated by a universal computing device. Such processes necessarily operate on finite representations.

Additionally, physical limits on information processing impose discreteness:

- Landauer's principle establishes a minimum energy cost for erasing a bit of information.
- Physical bounds on information density constrain the number of distinguishable states in any finite region.

Therefore, both computational and physical considerations support the discreteness of realizable state spaces.

9 Emergence of Logical Systems

Logical systems arise as constructions over discrete distinctions:

Logical systems arise as constructions over discrete distinctions. Their differences correspond to different structural organizations of distinctions:

Logic	**Structure over Distinctions**
Classical	Equivalence partition of distinctions
Fuzzy	Ordered aggregation of distinctions
Modal	Distinctions across indexed domains
Intuitionistic	Constraints on construction sequences

In each case, the logical system does not introduce new ontological primitives, but reorganizes the same underlying discrete distinctions. Thus, non-classical logics are not fundamentally new ontologies, but structured transformations of discrete distinctions.

10 Continuum as Limit Representation

Continuous domains such as $[0, 1]$ assume infinite divisibility. Under the present framework:

$$[0, 1] = \lim_{N \to \infty} D_N$$

where D_N is a finite discrete structure.

The limit preserves relational structure (such as order), but introduces infinite density, which is not realizable under the axiom of discrete distinguishability.

Thus, the continuum should be understood as a compact representation of increasingly fine discrete structures, rather than an ontological entity. Thus, continuity is a limit construction, not an ontological primitive.

11 Consequences

- Logic is grounded in distinguishability, not truth values.
- Discreteness is ontologically primary.
- Continuous models are secondary representations.
- Logical systems reflect organization of distinctions.

12 Minimality of the Framework

We argue that the proposed framework is minimal with respect to the construction of logical systems.

12.1 Sufficiency

The primitives introduced:

- a set of states S,
- a distinction operation δ,
- an order relation $\prec$,
- composition operations,

are sufficient to construct:

- logical operations (negation, conjunction, disjunction, implication),
- semantic structures (interpretation and satisfaction),
- model-theoretic structures,
- computational representations,
- various classes of logical systems.

12.2 Irreducibility

Each primitive plays an essential role:

- Without δ, no distinctions can be formed.
- Without $\prec$, implication and structure of reasoning cannot be defined.
- Without composition, complex structures cannot be constructed.

Thus, removal of any primitive prevents construction of general logical systems.

12.3 Non-Redundancy

No primitive can be derived from the others:

- Order $\prec$ is not reducible to δ, as distinction does not determine sequence.
- Distinction δ is not reducible to order, as ordering presupposes distinguishability.
- Composition is not reducible to either distinction or order, as it introduces structural closure.

12.4 Finite Realizability Constraint

The restriction to finite realizability does not introduce additional primitives, but constrains admissible structures:

- all realizable systems are finite or finitely representable,
- infinite constructions arise only as limits of finite systems.

12.5 Conclusion

The framework $(S, \delta, \prec)$ with composition constitutes a minimal and sufficient basis for the construction of logical systems. No additional primitives are required.

13 Conclusion

We have proposed a discrete foundation of logic grounded in distinguishability and order. In this framework, the fundamental unit of logic is not a truth value, but an act of distinction embedded in a directed structure.

Starting from minimal ontological primitives, we constructed a distinction algebra in which logical operations arise as derived structures from composition, compatibility, and ordering. Classical and non-classical logical systems were shown to emerge as specific configurations of these underlying relations, rather than as independent formal systems.

The framework eliminates the need for primitive truth assignments. Logical semantics is reconstructed in terms of realizability of distinction structures, and models are defined intrinsically, without reference to external valuation functions. This yields a unified account of syntax, semantics, and computation within a single formal system.

Continuous domains are reinterpreted as limit representations of discrete structures. Under the constraint of finite realizability, only discretely distinguishable states are ontologically admissible, while infinite constructions remain representational tools.

The resulting framework is both sufficient and minimal: all major logical constructions can be derived from distinction and order, and no additional primitives are required. This establishes a unified discrete basis for logic consistent with physical and computational constraints.

More broadly, the work reframes logic not as a system of truth-bearing expressions, but as a theory of structured distinctions. Logical systems become organized configurations of distinguishability, and their differences correspond to variations in order, compatibility, and composition.

A Applications to Non-Classical Logics

This appendix outlines how selected classes of logical systems arise as specific configurations of distinction algebra.

A.1 Classical Logic

Classical logic is obtained under the following conditions:

- The order $\prec$ is total.
- Every distinction admits an inverse: $\forall d \in D,\ \exists d^{-1}$.
- Composition is deterministic and closed.

Under these constraints, distinctions can be partitioned into two equivalence classes:

$$D = D_T \cup D_F$$

Logical operations reduce to operations over this binary partition.

A.2 Constructive Logic

A constructive logical system arises when:

- The order $\prec$ is partial.
- Not all distinctions admit inverses.
- Implication reflects constructive reachability:

$$d_1 \Rightarrow d_2 \text{ iff } d_1 \prec d_2$$

Negation is not necessarily involutive, reflecting the absence of global inversion.

A.3 Graded Logic

A graded logical system arises when:

- Distinctions are aggregated into an ordered scale.
- The order $\prec$ is extended to a linear ordering.
- Logical operations are induced by monotonic transformations over the order.

A normalized representation may be introduced:

$$\mu(d_k) = \frac{k}{n}$$

A dense scale appears as a limit representation:

$$\lim_{n\to\infty} D_n$$

A.4 Modal Logic

A modal logical system arises when distinctions are organized across multiple domains:

- Let $D = \bigcup_i D_i$ be a family of distinction sets.
- Each D_i has its own order $\prec_i$.
- Cross-domain relations define accessibility:

$$R \subseteq D_i \times D_j$$

Modal operators correspond to transitions between domains of distinctions.

A.5 Non-Explosive Logic

A non-explosive logical system arises when:

- Incompatibility is not globally enforced.
- Distinctions d and $\neg d$ may coexist.
- Composition rules restrict propagation of contradiction.

Thus, contradiction corresponds to coexistence of incompatible distinctions without collapse of the system.

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